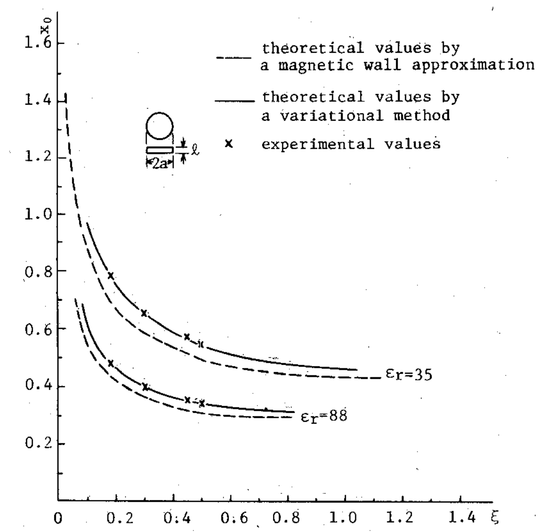

Fig. 3 Comparison between theoretical values and experimental ones.

variational method agree well with the experimental ones to within an error of 1 percent.

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Reciprocity Theorem for a Region with Inhomogeneous Bianisotropic Media and Surface Impedance

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***Abstract*—It is shown that the modified reciprocity theorem holds for a region bounded by inhomogeneous anisotropic impedance surfaces and composed of regions with lossless inhomogeous bianisotropic media; and besides, the reciprocity theorem holds for the case with a condition.**

I. INTRODUCTION

Bianisotropic media or moving media have been treated in the literature [1]–[13], [18]. A moving medium, even if it is isotropic in its rest frame, must be treated as bianisotropic [6]. Recently, it was shown by Kong and Cheng [1] that a properly modified reciprocity theorem could be applied to bianisotropic media; they discussed the modified reciprocity theorem between a single-bianisotropic medium region and its complementary region. These regions are either bounded by perfectly conducting surfaces, or by surfaces that recede to infinity. However, waveguides with surface impedance walls are also of interest [14], [15].

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In this short paper, the modified reciprocity theorem is derived in a region bounded by impedance surfaces; and besides, the reciprocity theorem is derived for the case with a condition.

II. THEORY

Consider a region R consisting of p subregions in a three-dimensional space. The region R is bounded by inhomogeneous anisotropic impedance surface S and is composed of the regions R_i ($i = 1,2,\cdots,p$). Let the region R_i be filled with a lossless inhomogeneous bianisotropic medium [3], [18] and be bounded by the surface S_i.

Maxwell's equations for a time-harmonic and finite electric current density, $\boldsymbol{J}(\boldsymbol{r}) \exp(j\omega t)$, in such a region R are given by

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = -j\omega \boldsymbol{B}(\boldsymbol{r}) \tag{1}$$

$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = j\omega \boldsymbol{D}(\boldsymbol{r}) + \boldsymbol{J}(\boldsymbol{r}) \tag{2}$$

subject to the constitutive relations

$$\boldsymbol{D}(\boldsymbol{r}) = \bar{\bar{\varepsilon}}(\boldsymbol{r})\cdot\boldsymbol{E}(\boldsymbol{r}) + \bar{\bar{\kappa}}(\boldsymbol{r})\cdot\boldsymbol{H}(\boldsymbol{r}) \tag{3}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \bar{\bar{\nu}}(\boldsymbol{r})\cdot\boldsymbol{E}(\boldsymbol{r}) + \bar{\bar{\mu}}(\boldsymbol{r})\cdot\boldsymbol{H}(\boldsymbol{r}) \tag{4}$$

and the impedance boundary condition

$$\hat{n} \times \boldsymbol{E}(\boldsymbol{r}) = -\hat{n} \times [\bar{\bar{Z}}(\boldsymbol{r})\cdot\{\hat{n} \times \boldsymbol{H}(\boldsymbol{r})\}] \quad \text{on } S \tag{5}$$

and the conditions at the interfaces $S_i \cap S_k$ ($i,k = 1,2,\cdots,p$; $i \neq k$)

$$\hat{n}_i \times \{\hat{n}_i \times \boldsymbol{E}(\boldsymbol{r})\}|_{S_i\cap S_k, \boldsymbol{r}\in S_i} = \hat{n}_k \times \{\hat{n}_k \times \boldsymbol{E}(\boldsymbol{r})\}|_{S_k\cap S_i, \boldsymbol{r}\in S_k} \tag{6}$$

$$\hat{n}_i \times \{\hat{n}_i \times \boldsymbol{H}(\boldsymbol{r})\}|_{S_i\cap S_k, \boldsymbol{r}\in S_i} = \hat{n}_k \times \{\hat{n}_k \times \boldsymbol{H}(\boldsymbol{r})\}|_{S_k\cap S_i, \boldsymbol{r}\in S_k} \tag{7}$$

where

$\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}, \bar{\bar{\kappa}}, \bar{\bar{\nu}}$	tensors of rank 2;
$\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}_i,\ \bar{\bar{\mu}} = \bar{\bar{\mu}}_i,\ \bar{\bar{\kappa}} = \bar{\bar{\kappa}}_i,\ \bar{\bar{\nu}} = \bar{\bar{\nu}}_i$	when $\boldsymbol{r} \in R_i$ ($i = 1,2,\cdots,p$);
$\hat{n}, \hat{n}_i$	outer normal unit vectors to S and S_i ($i = 1,2,\cdots,p$), respectively;
$\bar{\bar{Z}}$	= surface impedance tensor.

Now let us assume that the medium is such that [3], [18]

$$\bar{\bar{\varepsilon}}^{*T} = \bar{\bar{\varepsilon}} \tag{8}$$

$$\bar{\bar{\mu}}^{*T} = \bar{\bar{\mu}} \tag{9}$$

$$\bar{\bar{\kappa}}^{*T} = \bar{\bar{\nu}} \tag{10}$$

and let the field solutions corresponding to current sources $\boldsymbol{J}_a$ and $\boldsymbol{J}_b$ be $\boldsymbol{E}_a$ and $\boldsymbol{H}_a$, and $\boldsymbol{E}_b$ and $\boldsymbol{H}_b$, respectively. We obtain

$$\nabla\cdot(\boldsymbol{E}_a^* \times \boldsymbol{H}_b + \boldsymbol{E}_b \times \boldsymbol{H}_a^*) = -\boldsymbol{J}_a^*\cdot\boldsymbol{E}_b - \boldsymbol{J}_b\cdot\boldsymbol{E}_a^* \tag{11}$$

where the asterisk denotes the complex conjugate and the superscript T denotes the transpose. Applying the divergence theorem to (11), we obtain the following modified Lorentz reciprocity theorem for the region R_i ($i = 1,2,\cdots,p$):

$$\iiint_{R_i} (-\boldsymbol{J}_a^*\cdot\boldsymbol{E}_b - \boldsymbol{J}_b\cdot\boldsymbol{E}_a^*)\, dV = \iint_{S_i} (\boldsymbol{E}_a^* \times \boldsymbol{H}_b + \boldsymbol{E}_b \times \boldsymbol{H}_a^*)\cdot\hat{n}_i\, dS. \tag{12}$$

By using (5), the integrand in the right-hand side of (12) becomes zero on the surface $S_i \cap S$ if the surface impedance tensor $\bar{\bar{Z}}(\boldsymbol{r})$, which is defined in (5), is such that

$$\bar{Z}^{*T} = -\bar{Z}. \tag{13}$$

Therefore, (12) becomes

$$\iiint_{R_i} (-J_a^* \cdot E_b - J_b \cdot E_a^*)\, dV$$

$$= \iint_{\Sigma_i(k\neq i)} (E_a^* \times H_b + E_b \times H_a^*) \cdot \hat{n}_i\, dS \tag{14}$$

where

$$\textstyle\sum_i (k \neq i) \equiv \bigcup_{k=1, k\neq i}^{p} S_i \cap S_k. \tag{15}$$

Summing (14) over all i and rearranging those surface integrals by using (6) and (7), we obtain

$$\iiint_R J_a^* \cdot E_b\, dV = -\iiint_R J_b \cdot E_a^*\, dV. \tag{16}$$

Now we define the quantities as follows:

$$\{a,b\} = j \iiint_R J_a^* \cdot E_b\, dV \tag{17}$$

$$\{b,a\} = j \iiint_R J_b^* \cdot E_a\, dV. \tag{18}$$

Then the complex conjugates of those quantities yield as follows:

$$\{a,b\}^* = -j \iiint_R J_a \cdot E_b^*\, dV \tag{19}$$

$$\{b,a\}^* = -j \iiint_R J_b \cdot E_a^*\, dV. \tag{20}$$

By using (16)–(20), we obtain the following modified reciprocity theorems:

$$\{a,b\} = \{b,a\}^* \tag{21}$$

$$\{b,a\} = \{a,b\}^*. \tag{22}$$

Let J_a and J_b be infinitesimal current elements as follows:

$$J_a = J_a' \exp(j\theta_a)\delta(r - r_a) \tag{23}$$

$$J_b = J_b' \exp(j\theta_b)\delta(r - r_b) \tag{24}$$

where J_a' and J_b' are real vectors. Let E_a and E_b be expressed as follows:

$$E_a = E_a' \exp[j\{\theta_a + \phi_a(r,r_a)\}] \tag{25}$$

$$E_b = E_b' \exp[j\{\theta_b + \phi_b(r,r_b)\}] \tag{26}$$

where E_a' and E_b' are real vectors of functions of position, and ϕ_a and ϕ_b are phase differences of E_a to J_a and of E_b to J_b, respectively.

Using (23)–(26), (16) becomes

$$J_a' \cdot E_b' \exp[j\{\theta_b - \theta_a + \phi_b(r_a,r_b)\}]$$

$$= J_b' \cdot E_a' \exp[j\{\pm\pi + \theta_b - \theta_a - \phi_a(r_b,r_a)\}]. \tag{27}$$

Then we obtain the following relations:

$$J_a' \cdot E_b' = J_b' \cdot E_a' \tag{28}$$

$$\phi_a(r_b,r_a) + \phi_b(r_a,r_b) = \pm\pi. \tag{29}$$

We consider a case in which the phase difference of E_a at the point r_b to J_a equals that of E_b at the point r_a to J_b, that is,

$$\phi_a(r_b,r_a) = \phi_b(r_a,r_b). \tag{30}$$

Therefore, we obtain from (29) and (30)

$$\phi_a = \phi_b = \pm\pi/2. \tag{31}$$

From (23) through (26), (28), and (31), we obtain

$$J_a \cdot E_b = J_b \cdot E_a. \tag{32}$$

In a case in which current sources are not infinitesimal current elements, the following reciprocity theorem holds:

$$\iiint_R J_a \cdot E_b\, dV = \iiint_R J_b \cdot E_a\, dV. \tag{33}$$

Using the reactions [16] and [17], (33) can be rewritten as follows:

$$\langle a,b\rangle = \langle b,a\rangle \tag{34}$$

where

$$\langle a,b\rangle = \iiint_R J_a \cdot E_b\, dV.$$

From (33), besides, we find that the following equation holds:

$$\iint_S (E_a \times H_b - E_b \times H_a) \cdot \hat{n}\, dS$$

$$= j\omega \iiint_R [E_a \cdot (\bar{\epsilon}^T - \bar{\epsilon}) \cdot E_b - H_a \cdot (\bar{\mu}^T - \bar{\mu}) \cdot H_b$$

$$+ H_a \cdot (\bar{\nu} + \bar{\kappa}^T) \cdot E_b - E_a \cdot (\bar{\nu}^T + \bar{\kappa}) \cdot H_b]\, dV. \tag{35}$$

Equations (31) and (34) enable us to state the following results if a medium is such [(8)–(10)] that the right-hand side of (12) vanishes on the closed surface S bounding the region R, and (30) holds.

1) The phase of an electric field leads or lags by $\pi/2$ more than that of a corresponding electric current source.

2) The reaction of one set of sources on another is equal to the reaction of sources of the latter set on the former.

3) Although reactions do not in general represent power, reaction is a useful quantity [17, p. 118]. Because of this conservative property [(34), for example], reaction can be used as a measure of equivalency for a region consisting of bianisotropic media and bounded impedance surfaces.

4) Equation (35) holds.

III. CONCLUSION

The modified reciprocity theorem holds for a region enclosed with an impedance surface; and besides, the reciprocity theorem holds for the case with a condition although the reciprocity theorem does not generally hold for a bianisotropic medium which is nonreciprocal. The system may be considered to be a resonator, where the electric and magnetic fields form standing waves, with the phase difference of electric field relative to the current source equal to $\pm\pi/2$.

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New Results in the Least pth Approach to Minimax Design

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***Abstract*—We present results of two general approaches for obtaining minimax designs through a sequence of least pth approximations demonstrating increased efficiency over previous least pth algorithms. Documented computer programs are available.**

INTRODUCTION

This short paper demonstrates the acceleration of convergence to minimax solutions by extrapolation on a sequence of least pth solutions [1] with geometrically increasing values of p, and compares the results with an efficient extension of work by Charalambous and Bandler [2], [3], in which a sequence of least pth solutions with finite values of p are obtained in an effort to reach a minimax solution. Documented computer programs are available [4], [5], as well as the theoretical background [6]–[8].

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THEORY

We minimize, with respect to ϕ for given ξ and $p > 1$, the function

$$U(\phi,\xi,p) \triangleq \begin{cases} M(\phi,\xi)\left(\sum_{i\epsilon K}\left(\dfrac{f_i(\phi)-\xi}{M(\phi,\xi)}\right)^q\right)^{1/q}, & M(\phi,\xi) \neq 0 \\ 0, & M(\phi,\xi) = 0 \end{cases} \quad (1)$$

where

$$M_f(\phi) \triangleq \max_{i\epsilon I} f_i(\phi), \qquad M(\phi,\xi) \triangleq M_f(\phi) - \xi,$$

$$q \triangleq p \operatorname{sgn} M(\phi,\xi)$$

and

$$K = \begin{cases} I \subset \{1,2,\cdots,m\}, & M(\phi,\xi) < 0 \\ J \triangleq \{i \mid f_i(\phi) - \xi \geq 0, i \in I\}, & M(\phi,\xi) > 0 \end{cases} \quad (2)$$

and where $\phi \triangleq [\phi_1\phi_2\cdots\phi_k]^T$ is the design parameter vector, and $f_1(\phi), f_2(\phi), \cdots, f_m(\phi)$ are m linear or nonlinear functions directly related to the response error functions such that if $M_f(\phi) > 0$ the specifications are violated and if $M_f(\phi) < 0$ the specifications are satisfied.

Charalambous has shown [6] that if we have $\boldsymbol{u}$ and ϕ such that

$$\sum_{i=1}^{m} u_i \nabla f_i(\phi) = \mathbf{0}$$

$$\sum_{1=1}^{m} u_i = 1, \qquad u_i \geq 0, \qquad i = 1,2,\cdots,m \quad (3)$$

then, if $\sum_{i=1}^{m} u_i f_i(\phi)$ is convex with respect to ϕ,

$$\sum_{i=1}^{m} u_i f_i(\phi) \leq M_f(\check{\phi}) \leq M_f(\phi) \quad (4)$$

where $\check{\phi}$ is the minimax optimum which is being sought,

$$\boldsymbol{u} \triangleq [u_1u_2\cdots u_m]^T$$

and

$$\nabla \triangleq [\partial/\partial\phi_1\partial/\partial\phi_2\cdots\partial/\partial\phi_k]^T.$$

The conditions (3) are satisfied at each optimum point $\check{\phi}(p,\xi)$ for a least pth objective function, yielding

$$\sum_{i=1}^{m} u_i f_i(\check{\phi}(p,\xi)) \leq M_f(\check{\phi}) \leq M_f(\check{\phi}(p,\xi)) \quad (5)$$

where, assuming K contains all critical sample points,

$$u_i = \frac{v_i}{\sum_{i\epsilon K} v_i} \quad (6)$$

$$v_i = \begin{cases} \left(\dfrac{f_i(\check{\phi}(p,\xi)) - \xi}{M(\check{\phi},\xi)}\right)^{q-1}, & i \in K \\ 0, & i \notin K. \end{cases} \quad (7)$$

The first term of (5), under the stated conditions, is a lower bound on $M_f(\check{\phi})$. It is, at any least pth solution, an optimistic indication of the ultimate minimax error to be expected for a particular design.